Comparison of micromechanics methods for effective properties of multiphase viscoelastic composites

L. C. Brinson, W. S. Lin
Mechanical Engineering Department, 2145 Sheridan Road, Northwestern University, Evanston, IL 60208, USA

Abstract

In this paper we examine the use of several popular micromechanics methods for determination of effective composite properties when all phases are viscoelastic. The elasticity-based Mori-Tanaka method and the finite element method of cells are used, both via implementation of the elastic-viscoelastic correspondence principle. The finite element technique considers two different periodic microstructures, square and hexagonal arrays, the results of which are compared with each other and the Mori-Tanaka predictions. The resultant effective properties for viscoelastic composites are determined at a wide range of frequencies and compared with transformed Hashin-Shtrikman bounds and Gibiansky-Milton bounds. The Mori-Tanaka method in the transformed domain is by far the simplest to implement and it is shown that the method replicates the major features of the storage and loss moduli of the composite, including location and magnitude of the double loss peaks from the glass-rubber transition of each phase. It is also illustrated that the Mori-Tanaka method predicts a nearly log-linear relationship between resultant property and volume fraction at a given frequency. Limitations of the Mori-Tanaka method for viscoelastic composites are highlighted. © 1998 Published by Elsevier Science Ltd. All rights reserved.

1. Introduction

The use of polymeric matrix composite materials and multiphase polymeric systems has been steadily increasing in recent years. The former materials are candidate materials for current civil infrastructure renovation owing to their strength-to-weight ratio, environmental durability and cost effectiveness; the latter materials are being further developed for the increased toughness they provide over homogeneous polymer materials. Consequently, knowledge of the effective properties of viscoelastic composites is of great interest. In many structural applications the viscoelastic nature of the properties becomes important and the long-term properties are required. To determine time-dependent properties of viscoelastic materials, accelerated testing is often performed in the laboratory (short-term tests at multiple temperatures) and time-temperature superposition is invoked to obtain the long-term response. For polymeric systems with more than one viscoelastic phase,* however, time-temperature superposition breaks down, precluding this avenue of material characterization. In such cases, numerical and analytical methods to determine material response at a full range of times become extremely important.

Micromechanics methods have been used with great success to predict effective moduli of elastic composite materials, given the properties of the component phases and the volume fractions. Schemes ranging from the Reuss and Voigt bounds, to the self-consistent method [6], to the method of cells [19], to finite element techniques, have been developed and applied extensively for elastic composites. See an excellent summary of the various techniques in the text by [1]. Comparison between the effective medium theories for elastic materials can be found in [12], with special attention to interface conditions. Direct comparison between effective medium theories and the unit cell finite element method, however, are largely absent from the literature. For elastic materials, one article examines a comparison between finite element, several micromechanics models and experimental data for unidirectional fiber composites ([18]).

Although methods to predict elastic properties of composites are well-established, extension to viscoelastic materials has not yet been well examined. Weng and co-workers [13,21] have applied the Mori-Tanaka method to viscoelastic materials, but focused on the
The finite element unit cell technique, while demonstrated to be accurate in reproducing experimental data, is cumbersome to apply. For each material system geometry, a new finite element mesh is required, and the unit cell boundary value problem must be solved separately at a full spectrum of frequencies. Consequently, use of a simpler algebraic technique to calculate effective viscoelastic properties accurately would be of great use. In this paper, the Mori-Tanaka method is extended via the correspondence principle to the frequency domain and the results for multiphase viscoelastic systems are compared with the finite element results for two different unit cell structures. The composite property results for all the methods are compared with bounding methods by Hashin-Shtrikman and Gibiansky-Milton to further assess the results. In the sections that follow, first the necessary background material is given for the constitutive response of the individual phases and the correspondence principle is outlined. Then the extension of the finite element technique and the Mori–Tanaka technique to the viscoelastic domain is briefly described. Finally, results of the methods are presented and compared with bounding methods.

2. Background

The time dependence of viscoelastic material response is often represented with a constitutive law in the form of a Stieltjes convolution. In this manner, the response of the material to loading history and the time dependence of the material properties are taken into account. In this work, it will be assumed that each component phase in the composite is an isotropic, linear viscoelastic material, in which case the constitutive law may be written

\[ s_{ij}(t) = \int_{-\infty}^{t} 2G(t-\zeta) \frac{de_{ij}}{d\zeta} d\zeta \]

\[ \sigma_{ik}(t) = \int_{-\infty}^{t} 3K(t-\zeta) \frac{de_{ik}}{d\zeta} d\zeta \]

(1)

Here, \( s_{ij} \) and \( \sigma_{ik} \) (\( e_{ij} \) and \( e_{ik} \)) are the deviatoric and dilatational components of the stress (strain) tensor and \( G(t) \) and \( K(t) \) are the time-dependent shear and bulk moduli respectively.

Owing to the hereditary integral expressions, determination of long-term material response for viscoelastic materials can become computationally intensive. Consequently, a tool commonly used to avoid this situation is a reformulation of the problem in the Fourier or Laplace domain via the elastic–viscoelastic correspondence principle ([5,7]). Imposition of time–harmonic deformation histories

\[ e_{ij}(t) = e_{ij}(\omega)e^{i\omega t} \]

reduces eqn (1) to

\[ \tilde{s}_{ij}(\omega) = 2i\omega \tilde{G}(\omega) \tilde{e}_{ij}(\omega) \]

\[ \tilde{\sigma}_{ik}(\omega) = 3i\omega \tilde{K}(\omega) \tilde{e}_{ik}(\omega) \]

(3)

where \( \tilde{G} \) and \( \tilde{K} \) are the half-sided Fourier transforms of the moduli, e.g.,

\[ \tilde{G}(\omega) = \int_{0}^{\infty} G(t)e^{-i\omega t} dt \]

(4)

and the complex moduli, where the storage and loss moduli are the real and imaginary parts respectively, are given by

\[ i\omega \tilde{G}(\omega) = G^*(\omega) = G'(\omega) + iG''(\omega) \]

\[ i\omega \tilde{K}(\omega) = K^*(\omega) = K'(\omega) + iK''(\omega) \]

(5)

Here and throughout, an overbar or * indicates a complex quantity with real and imaginary parts, which is a function of frequency.

In the transformed domain, the hereditary integrals of the viscoelastic constitutive law have reduced to expressions identical to that of elasticity (eqn (3)), but where now all field quantities are complex and functions of frequency. This technique allows for many solutions developed in elasticity to be used for viscoelastic problems. In the subsequent sections, the correspondence principle is applied to the Mori–Tanaka method and the finite element unit cell method to predict the effective properties of viscoelastic composites. In these cases, the results are left in the form of the complex moduli (rather than transformed back to the time domain), since it is often performed to characterize polymeric materials, yielding the complex moduli directly.

The effective properties determined here are the 2-D properties, representative of the transverse behavior of fibrous composites. Thus a two-dimensional plane-strain finite element analysis is used and the corresponding plane strain moduli are determined via the Mori–Tanaka method. The data used for individual phases of the composite are two idealized viscoelastic systems with glassy and rubbery asymptotic moduli and a broad range of relaxation times. The relaxation times and moduli are
chosen such that one material is at all times stiffer than the other material and the loss peaks from the two phases do not coincide. These features will aid in interpretation of the results. The properties of the phases are indicated on Fig. 3 in the Results section and can be described by Prony series of the form

\[ G(t) = G_* + \sum_{j=1}^{N} G_j e^{-\omega j} \]  

(6)

where \( G_* \) is the rubbery asymptotic modulus and \( G_j \) and \( \xi_j \) are the relaxation moduli and relaxation times. The Prony series terms for the two phases are given in Table 1. Note that the half-sided Fourier transform of eqn (6) yields the complex modulus in closed form:

\[ G^*(\omega) = \sum_{j=1}^{N} \frac{G_j \omega^2}{(1/\xi_j) + \omega^2} \]

\[ G^*(\omega) = G_* + \sum_{j=1}^{N} \frac{G_j \omega^2}{(1/\xi_j) + \omega^2} \]

\[ = G^*(\omega) + iG^\prime(\omega) \]  

(7)

3. Finite element unit cells

In the finite element method, the composite is considered to have a periodic microstructure, which can be subdivided into appropriate unit cells. The behavior of the cell boundaries when the overall composite is subject to appropriate prescribed loading provides the boundary conditions on these unit cells, such that consideration of a single unit cell simulates the behavior of the entire composite. Effective moduli are then obtained by solving the boundary value problem on the unit cell and interpreting the results. The application of the correspondence principle to the finite element method was thoroughly described in Brinson and Knauss [4,5] and will not be reiterated here. The externally apparent changes to the modified finite element code are the input of the viscoelastic properties of the individual phases, the complex form used for the specification of boundary conditions and for the field quantities in the solution, and the necessity to solve the problem separately for each frequency of interest. Typically, the problem is solved at regular intervals in log-frequency space to provide the full spectrum of material response.

For example, consider the square array of inclusions in Fig. 1(a) as the composite material, subjected to a uniform tensile load in the \( y \)-direction. Owing to symmetry of the material and the loading, a unit cell, shown in Fig. 1(b) with boundary conditions, is sufficient to determine overall composite response. Similarly, for a hexagonal array of inclusions, Fig. 2(a), the unit cell in Fig. 2(b) is used. Given the displacement in the \( y \)-direction, the resultant average stress in the \( y \)-direction (obtained by a nodal average along \( y = L \)) can be used to determine the effective plane-strain Young's modulus \( E^* \) of the composite

\[ \sigma_{\text{avg}}(\omega) = \frac{1}{L^2} \int_0^L \int_0^L \sigma_{yy}(x,y) \, dx \, dy \]  

\[ = \frac{1}{L^2} \int_0^L \int_0^L \sigma_{yy}(x,y) \, dx \, dy \]

\[ = \sigma_{yy}(x,y) \]

\[ = \sigma_{yy}(x,y) \]

\[ = \sigma_{yy}(x,y) \]

\[ = \sigma_{yy}(x,y) \]

\[ = \sigma_{yy}(x,y) \]  

(8)

Similarly, the in-plane, plane-strain Poisson's ratio \( v^* \) for the composite can be determined by examining the resultant displacement along \( x = L \) (\( x = 3L \) for a hexagonal array), required to maintain \( \sigma_{\text{avg}}, \epsilon_{\text{avg}} = 0 \).

\[ v^* = \frac{\epsilon_{yy}(x=L)}{\epsilon_{xx}(x=L)} \]  

(9)

The plane-strain bulk modulus (used in later comparisons with bounding theorems) can be obtained from

\[ K^*(\omega) = \frac{E^*(\omega)}{2(1-v^*)} \]  

(10)

4. Mori–Tanaka method

The Mori–Tanaka method is an extremely powerful effective medium theory, described originally by Mori and Tanaka [16] and more recently reformulated and clarified by Benveniste [2]. The method is attractive owing to both its simplicity in application and its versatility in accommodating a variety of composite conditions. The use of the Eshelby tensor to specify inclusion shape and orientation allows consideration of composites with inclusions ranging from cylindrical fibers, to spherical particulates, to oblate and penny-shaped phases. Extensions to the Mori–Tanaka method can accommodate randomly oriented [20] as well as aligned inclusions, and multiple included phases [22], as well as coated inclusions for study of interlayers [3].

This method has been widely applied for elastic phases and the extension to viscoelastic media via the correspondence principle is straightforward, simply substituting complex moduli in place of elastic moduli in the final expressions. The resultant effective complex modulus \( L^* \) for a two-phase composite can be given most generally by

\[ L^* = L_f^* \left( I - c_1 (S^* c_o + c_1 I) \right) \]

\[ + \left( L_i^* - L_f^* \right)^{-1} \left( \lambda \right)^{i} \]  

(11)

where \( L_f^* \) and \( L_i^* \) are the complex moduli of the matrix and inclusion materials, \( c_0 \) and \( c_1 \) are the volume fractions of matrix and inclusion, \( S^* \) is the Eshelby tensor (as given in Mura [17], with complex Poisson ratio of the matrix \( v^* \) replacing \( v_0 \)). Note that eqn (11)
Fig. 3. Comparison of (a) storage and (b) loss Young’s modulus for composites with volume fractions 36% stiff material, 64% soft material.
Table 1.
Prony series terms for moduli of the two phases considered in the composite

<table>
<thead>
<tr>
<th>Stiff material shear moduli $G^{(1)}$</th>
<th>Soft material shear moduli $G^{(2)}$</th>
<th>Bulk and asymptotic moduli</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G^{(1)}$</td>
<td>$G^{(2)}$</td>
<td>$G^{(1)}$</td>
</tr>
<tr>
<td>3</td>
<td>3.162</td>
<td>0.032</td>
</tr>
<tr>
<td>10</td>
<td>17.783</td>
<td>0.100</td>
</tr>
<tr>
<td>32</td>
<td>100.000</td>
<td>0.316</td>
</tr>
<tr>
<td>100</td>
<td>316.228</td>
<td>1.000</td>
</tr>
<tr>
<td>316</td>
<td>1000.000</td>
<td>3.162</td>
</tr>
<tr>
<td>1000</td>
<td>5623.413</td>
<td>10.000</td>
</tr>
<tr>
<td>3162</td>
<td>10000.000</td>
<td>31.623</td>
</tr>
<tr>
<td>10000</td>
<td>5623.411</td>
<td>100.000</td>
</tr>
<tr>
<td>31623</td>
<td>141.254</td>
<td>316.228</td>
</tr>
<tr>
<td>1000000</td>
<td>56.234</td>
<td>1000.000</td>
</tr>
<tr>
<td>316228</td>
<td>17.783</td>
<td>3162.278</td>
</tr>
<tr>
<td>1000000</td>
<td>5.623</td>
<td>10000.000</td>
</tr>
<tr>
<td>3162278</td>
<td>3.162</td>
<td>31622.777</td>
</tr>
<tr>
<td>10000000</td>
<td>1.778</td>
<td>100000.000</td>
</tr>
</tbody>
</table>

Fig. 1. (a) Periodic microstructure: square array. Lines of symmetry shown. (b) Unit cell from square array: boundary value problem for finite element analysis illustrated. Note $\delta$ determined such that the average tractions on the surface are zero.

Fig. 2. (a) Periodic microstructure: hexagonal array. Lines of symmetry shown. (b) Unit cell from hexagonal array: boundary value problem for finite element analysis illustrated.
produces the three-dimensional moduli, so appropriate transformation must be made on the results to compare with the plane-strain moduli of the previous section. For a transversely isotropic fibrous composite, it can be easily shown that the plane-strain, transverse Young's and bulk moduli and the plane-strain, in-plane Poisson's ratio are given from the components of eqn (11) by

\[ L^* = \frac{(L_{53}^*)^2 - (L_{55}^*)^2}{L_5} \quad (12) \]

\[ K^* = \frac{2(L_{52}^* + L_{53}^*)}{L_5} \quad (13) \]

\[ v^* = \frac{L_{53}^*}{L_{52}^*} \quad (14) \]

The viscoelastic domain problem was recently studied by Weng and co-workers \([13,14,23]\), and complete expressions for a variety of inclusion shapes are provided in those references. The results consider cases of viscoelastic matrices with elastic inclusions, where the matrix is assumed to have simple Maxwell-type behavior. In the present work, longer time responses are considered, both phases are viscoelastic and the individual phases possess a broad range of relaxation mechanisms, as indicated by the number of terms in the Prony series expansion.

The assumptions leading to the property predictions for the Mori–Tanaka method are such that the overall composite is globally isotropic. Whereas this condition is also met by the hexagonal array for the periodic microstructures in the finite element technique, it is not met by the square array. Since no information is provided to the Mori–Tanaka method of material microgeometry, it is typically assumed to be most representative of a composite with inclusions of random size, randomly dispersed.

5. Results

In this section we present a prediction of composite properties using the idealized viscoelastic properties from the Prony series given in Table 1. Results for the complex moduli as a function of frequency are presented for two periodic arrays using the finite element method and a random microstructure using the Mori–Tanaka method. All results presented are for the 2-D plane strain moduli of the two-phase composite. The differences between the predictions of the methods are highlighted and results are compared with analytical bounds.

Since the finite element unit cell method was shown to be able to reproduce experimental data for an SBS polymeric system with a known periodic microstructure, the results from the finite element method in this work are assumed to represent accurately the overall properties of the respective periodic microstructure arrangements. Some conclusions can be drawn from these results about the accuracy of the Mori–Tanaka method to predict the composite response for these microstructures and the sensitivity of a viscoelastic composite to its microstructure.

In Fig. 3(a,b), an array of results for the transverse Young's modulus is presented to illustrate the variability of the composite properties across the full frequency range as a function of composite microgeometry. The results are for a constant phase fraction of 36% stiff material, and finite element predictions for composites with the stiff material in the inclusion and in the matrix are given; both square and hexagonal arrays are considered. The properties of the individual phases are included on the figures for comparison purposes. From these results, one can loosely state that the composite response is dominated by the matrix material properties: the magnitude and shape of the composite moduli as a function of frequency are generally closer to that of the matrix phase than the included phase. However, there is a large discrepancy between the square array and the hexagonal array microstructures, particularly in the storage modulus for the stiff matrix case. In all cases, the square array provides a stiffer response than the hexagonal array at the same volume fraction of inclusion. As will be shown later, the discrepancy between the two microstructures becomes more pronounced with increased volume fraction of the included phase. Voigt and Reuss bounds are also indicated on the plots and the two cases correspond, as expected, with results similar to the stiff matrix and soft matrix cases respectively.

One exception to the statement that the matrix material tends to dominate composite response, is the loss modulus at high frequency. Here the composite behavior for all configurations is clearly dominated by the loss modulus of the soft material, regardless of material microstructure. This result holds for a wide range of volume fractions of the two phases. In an earlier paper [5], it was speculated that the cause was an interaction between the high frequency (short time) of the calculations and the time dependence of the phase properties, reasoning that low losses in the stiff phase of the composite could not prohibit dissipation in the soft phase at high frequencies. To test this hypothesis, a simple study was performed here: the relaxation times of the moduli of the stiff material were all uniformly shifted by five decades towards shorter times (higher frequencies) to create an alternate stiff material. Then the results for composite properties were examined with the alternate stiff material in the matrix and the inclusion. The result for the loss modulus is illustrated for two cases in Fig. 4. These results clearly show that at high frequencies the composite response is no longer dominated by the
softer material; however, this is true instead at low frequencies. From analysis of the results (compare for example Fig. 3(b) and Fig. 4), it becomes clear that the loss modulus of the composite is completely dominated by the properties of one phase of the composite, say phase 2, if the following is true: $E_2' < E_1'$ and $E_2'' > E_1''$. These results are consistent for all methods of composite property prediction and (as will be seen later) are even reflected in the location of the bounds for the composite properties. This result indicates that, with proper choice of material properties, it is possible to engineer a multiphase polymer system to have a high loss modulus (good energy dissipation characteristics) for a wide range of frequencies without substantially degrading the stiffness of the composite (storage modulus).

To examine the effect of phase fraction on the composite response, as predicted by the finite element method and the Mori–Tanaka method, results were calculated for a wide range of volume fractions. Two sets of representative results are shown here in Figs 5 and 6. Fig. 5 shows a composite with 50% stiff matrix and predictions from all three methods. Note that in this case, the results of all methods are relatively close at all frequencies for both storage and loss moduli. In contrast, for 50% soft matrix (Fig. 6) there is a relatively large discrepancy between the prediction for square and hexagonal arrays, whereas the Mori–Tanaka and hexagonal array predictions are reasonably close. In most volume fractions considered (see Lin [15] for a complete exposition), the Mori–Tanaka predictions are closer to the hexagonal array prediction than the square array results, and in many cases the Mori–Tanaka method provides a good prediction of the hexagonal array response. Since the hexagonal array microstructure and the assumptions leading to the Mori–Tanaka method both result in an isotropic composite, the proximity of these predictions is reasonable.

An illustrative way to view the influence of phase fraction is shown in Fig. 7. Here, the response of the composite storage modulus at a given frequency is plotted versus volume fraction. Results for both stiff matrix and soft matrix cases are given, appearing in two different clusters on the figure. Note the striking result that as the volume fraction of inclusion increases, the discrepancies between the methods increases; this effect is most pronounced in the case of a soft matrix with stiff inclusions. From these and similar results at other frequencies, it can be concluded that the square array model provides the stiffest modulus in all cases, whereas the Mori–Tanaka method generally predicts the lowest values. In all cases, the Mori–Tanaka prediction is nearly linear with
Fig. 5. Comparison of (a) storage and (b) loss Young's moduli for composite with 50% soft inclusion.
Fig. 6. Comparison of (a) storage and (b) loss Young's moduli for composite with 50% stiff inclusion.
To analyze the composite property predictions further, comparisons can be made with several analytical bounding methods. Hashin–Shtrikman provide bounds on the properties of two-phase composites for elastic materials [10]; these can be transformed via the correspondence principle to provide bounds on the moduli of viscoelastic materials in the frequency domain [5,11]. However, as will be seen in Fig. 10, although the transformed bounds are valid in the transformed domain, the simple splitting those bounds into real and imaginary parts as done here provides only pseudobounds for the storage and loss moduli. Results for a 36% stiff material composite are illustrated in Fig. 9(a,b). In general, upper bounds are similar to predictions from the various methods for composites with stiff material in the matrix, whereas lower bounds are similar to composites with soft matrices. The square array predictions lie outside the bounds for the stiff matrix case, a discrepancy due to the anisotropic nature of the material microstructure in this case. Note that the results of the Mori–Tanaka method for the stiff matrix and soft matrix composites coincide with the upper and lower bounds respectively. For elastic materials, this result was proven by Weng [23]. Finally, note the very narrow gap between the upper and lower bounds in the high frequency loss modulus, representative of the result shown earlier that the composite loss moduli are dominated by the soft material alone at high frequencies (since $E''_s < E''_i$ and $E''_c > E''_i$ at high frequencies for this system).

*With the exception of the high frequency loss modulus, where the composite loss modulus is higher than that of either phase alone.
Gibiansky and Milton [9] have recently developed more rigorous bounds for the bulk moduli of viscoelastic composites. Figure 10 illustrates the distinction between the extension of Hashin–Shtrikman bounds to the viscoelastic domain and the Gibiansky–Milton bounds. At a given frequency, Hashin–Shtrikman (H–S) bounds provide upper and lower bounds on $K^*(\omega)$ (or $K(s)$ in the Laplace domain) [25]. To extend these rigorously to create bounds on the storage and loss moduli separately is not a trivial task, but has been accomplished by Milton and coworkers. Before examining these rigorous bounds, consider a simple (and obvious) approximation: for pseudobounds on the storage modulus use the real parts of the H–S upper and lower bounds; for pseudobounds on the loss modulus use the imaginary parts of the H–S upper and lower bounds. (This was done to obtain the bounds shown in Fig. 9.) This technique corresponds to the square shaded region in the complex plane shown in Fig. 10. The rigorous Gibianski–Milton bounds, however, define arcs in the complex plane at a given frequency which enclose the possible values of the complex modulus in terms of real and imaginary compounds directly. These bounds are based on the H–S upper and lower bounds and on the Reuss bound (harmonic average), the outermost pair of the three arcs define the bounds. As demonstrated in Fig. 10, in some cases the G–M bounds lie partially outside the approximate H–S pseudobounds. This reveals that although the H–S bounds are valid in the transformed domain, a simple split of the bounds into real and imaginary parts is not valid and more rigorous techniques [9,24] should be used. Thus the upper and lower bounds shown in Fig. 9 are not strict bounds, although they are good approximations to bounds. Figure 11 shows the Gibiansky–Milton bounds applied to a 36% stiff material composite at the given frequency. Note that since these bounds apply to the bulk modulus, all results practically coincide with the endpoints of the arcs, which are also the Hashin–Shtrikman upper and lower bounds. Also indicated in the figure are two data points for a layered inclusion case, in which the finite element method was used to predict the modulus of a composite where the inclusion was surrounded by a layer of matrix material, another layer of inclusion material, and then embedded in the composite. This case is included simply to illustrate that for a significantly changed microstructure, the property predictions for bulk modulus diverge from the other cases shown, and still lie within the bounds. Li and Weng [13] have examined cases of different inclusion shape and shown that those also lie within the Gibiansky–Milton bounds.

Gibiansky and Lakes [8] have recently provided bounds on the shear moduli of viscoelastic composites; their theory is restricted to cases of real and equal Poisson's ratios, which substantially changes the behavior of the composite. In those cases, the shear
Fig. 9. Comparison of (a) storage and (b) loss Young's moduli for composites with 36% stiff material with the Hashin-Shtrikman bounds.
modulus behaves similar to the bulk modulus for the cases considered here and no longer shows a large discrepancy between the various predictive methods. See for example Fig. 12.

1 A more recent paper [24] contains rigorous bounds for the shear modulus without such restrictions, and these bounds will be used in future work.

6. Conclusion

In this paper we have compared use of the Mori-Tanaka and unit cell finite element methods for prediction of viscoelastic composite properties. Two different periodic microstructures were considered in the finite element case. The results illustrate that while all methods predict the same gross trends in composite behavior, there are significant differences between the different predictive methods as a function of frequency.
Fig. 12. Gibiansky-Lakes bounds for shear modulus; real and equal Poisson ratios used for the individual phases as required by bounds.

as well as a function of volume fraction. The results for the isotropic composites all fall within several analytical bounding methods, providing confidence in the results, whereas the square array microstructure (anisotropic) falls outside the bounds for the stiff matrix case. In general, the square array microstructure is shown to exhibit a stiffer response than the hexagonal array microstructure or a random microstructure.

Since the magnitudes of the phase moduli change relative to one another with frequency, it can be anticipated that the discrepancies between the different methods will change with frequency, with more noticeable differences when the moduli of the phases are farther apart. However, in addition to this effect, at the frequency of the glass-rubber transition of the included phase, the composite predictions from the Mori–Tanaka and the hexagonal array finite element methods often reflect the property drop with a significant dip in the composite storage modulus, whereas the square array does not. Also, under special conditions of the phase moduli (met at high frequency in the case here) the loss modulus is completely dominated by the loss modulus of one phase, nearly independent of volume fraction and microgeometry of the composite. This latter result has implications for design and fabrication of high damping materials.

The discrepancy between the predicted effective moduli as a function of volume fraction is not unexpected, as this result is well documented for various effective medium theories with an elastic composite. Here we illustrate that with increasing volume fraction of inclusion the predictions of the different methods diverge quite sharply, especially in the case of a soft matrix with a stiffer inclusion. It is also shown that the Mori–Tanaka method predicts a nearly log-linear relationship of modulus magnitude with volume fraction.

In spite of the discrepancies between the methods for viscoelastic composites, it is noted that the Mori–Tanaka method is far easier to implement than the finite element technique: different inclusion shapes and orientations, as well as different inclusion properties, can be quickly and efficiently examined. This speed of calculation makes feasible the optimal design of microstructure and phase properties to provide superior composite response at all times and temperatures of interest. Given the results above, the Mori–Tanaka technique could be used with reasonable accuracy even for periodic microstructures if the volume fraction of inclusions are moderate. In this light, extension of the Mori–Tanaka results to examine more difficult problems in viscoelastic composites, such as interphase effects [3], is promising. The advantage of the finite element technique over that of the Mori–Tanaka method (or other effective medium theories) is that additional information, such as stress contours with respect to the microstructure, is calculated simultaneously, and further calculations, such as to examine fracture behavior, are straightforward extensions.

Acknowledgements

The authors appreciate partial support from NSF grant CMS-9501792 for this work.

References


